Concursul interjudețean de matematică
’’Memorialul Alexandru Cojocaru’’
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## Clasa a VIII a

1. a) Sa se determine multimile $D \subseteq R$ astfel incat functiile f,g : D $\to$ R, f(x)= $x^3 + x^2 + 1$ si g(x)=2x+1sa fie egale
   f(x)=g(x) $\Leftrightarrow$
   $x^3 + x^2 + 1 = 2x + 1 \Leftrightarrow x^3 + x^2 - 2x = 0 \Leftrightarrow x(x^2 + x - 2) = 0 \Leftrightarrow x(x-1)(x+2) = 0 \Leftrightarrow x \in \{-2,0,1\}$
   (1p) (1p)

D$\in$ {{-2,0,1},{-2,0},{-2,1},{0,1},{-2},{0},{1}}….(1p)

   b) Fie n un numar natural impar. Demonstrati ca fractia $\dfrac{n^3 + 2n^{2+3n+4}}{n^2+2}$ este ireductibila.

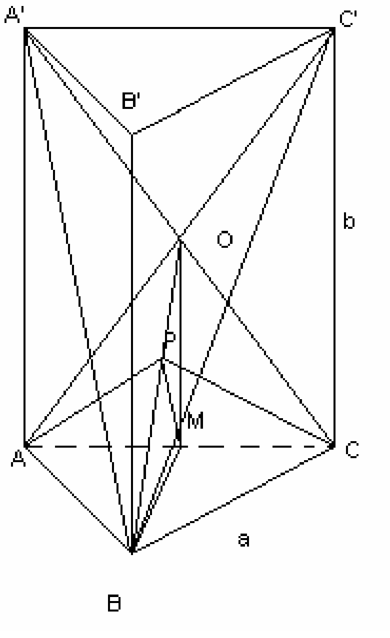
Fie ( $n^3 + 2n^2 + 3n + 4, n^2 + 2$ )=d $\Rightarrow \left.\begin{matrix} d/\ \ n^3 + 2n^2 + 3n + 4 \\ d/\ \ n^2 + 2 \Rightarrow d/\ \ n^3 + 2n \end{matrix}\right\} \Rightarrow \begin{matrix} d/\ \ 2n^2 + n + 4 \\ d/\ \ n^2 + 2 \Rightarrow d/\ \ 2n^2 + 4 \end{matrix} \Rightarrow$

$d/\ \ n \Rightarrow \begin{matrix} d/\ \ n^2 \\ d/\ \ n^2 + 2 \end{matrix} \Rightarrow d/\ \ 2$ ..(2p)

=>d=1(1p)

$\left.\begin{matrix} d/n^2 + 2 \\ n\_impar => n^2 + 2\_impar \end{matrix}\right\}$=>d impar (1p)

2. Se considera prisma triunghiulara regulata ABCA’B’C’. Sa se demonstreze ca planele (ABC’) si (BCA’) sunt perpendiculare, daca si numai daca AA’=AB $\frac{\sqrt{6}}{2}$.




Fie {O}=AC’ $\cap$ $A'C$ si OM $\perp$ AC (1p)

AC $\perp$ (BOM) (1p)

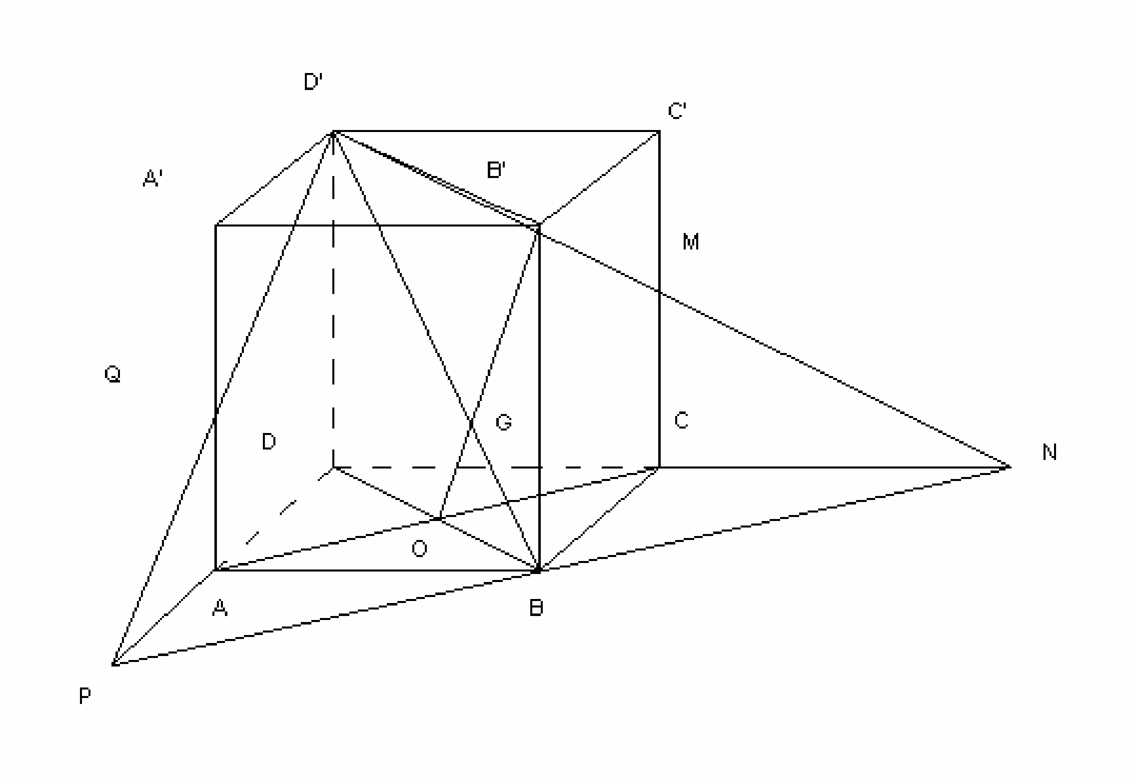
Fie MP $\perp$ OB (1p)

$\widehat{(ABC'),(BCA')} = \widehat{APC}$ (1p)

$\widehat{APC} = 90°$ (1p) $\Leftrightarrow$ MP= $\frac{AC}{2}$ $\Leftrightarrow$ $\frac{OM * BM}{OB} = \frac{AC}{2}$ (1p) $\Leftrightarrow$

$\Leftrightarrow \dfrac{\frac{b}{2} * \frac{a\sqrt{3}}{2}}{\frac{\sqrt{b^2 + 3a^2}}{2}} = \frac{a}{2} <=> b = \frac{a\sqrt{6}}{2}$ (1p)

3. Fie cubul ABCD A'B'C'D' si M mijlocul medianei CC'. Planul (BMD') intersecteaza DC si DA in N respectiv P. a) Demonstrati ca triunghiurile D'NP si ACB' au acelasi centru de greutate. b) Stabiliti raportul volumelor determinate de planul (MBD') in cubul ABCDA'B'C'D' si studiati cum se modifica acest raport daca $M \in CC'$ dar nu este mijlocul segmentului CC'.




a.Daca notam muchia cubului cu a
DN=2a, DP=2a….1p.
N,B,P coliniare, BP=BN$\Rightarrow$D'B mediana DD'PN…1p.
BD$\cap$AC={o}, BD $\cap$B'D={G} B'O mediana triunghi B'AC ….1p.
triungiul OBG~triunghi B'D'G $\Rightarrow \dfrac{OG}{GB'} = \dfrac{BG}{GD'} = \dfrac{1}{2} \Rightarrow$ G centr de greutate coincid …..1p.

b. Fie D'P$\cap$AA'={Q}$\Rightarrow V_{D'QABCMD}=V_{D'DPN}-V_{QAPB}-V_{MCBN}=\dfrac{a^3}{2} \Rightarrow$raportul=1….1p.

-daca $M \in$ [CC'] demonstratie identica….1p.
-daca $M \in$ CC' si $M \notin$ [CC'] planul taie alte doua muchii opuse ca la punctul anterior deci demonstratia nu se modifica…….1p.