Concursul interjudețean de matematică
’’Memorialul Alexandru Cojocaru’’
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Clasa a VII-a

1. a)Demonstrati ca $\sqrt{35n^2+42n+10} \in$ R – Q, oricare ar fi n numar rational.

$35n^2+42n+10=7(5n^2+6n+1)+3$……………………………………………………….1p

a$\in \{7k, 7k+1, 7k+2, 7k+3, 7k+4, 7k+5, 7k+6\}$, daca a $\in$ N

$(7k)^2=49k^2=7p$

$$\left.\begin{array}{l}(7k)^2=49k^2=7p\\(7k+1)^2=49k^2+14k+1=7(7k^2+2k)+1=7p+1\\(7k+2)^2=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots=7p+4\\(7k+3)^2=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots=7p+2\\(7k+4)^2=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots=7p+2\\(7k+5)^2=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots=7p+4\\(7k+6)^2=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots=7p+1\end{array}\right\}\Rightarrow \qquad 2p$$

$p\in$ N

$a^2\in\{7p, 7p+1, 7p+2, 7p+4 \mid p\in N\}\Rightarrow 7p+3\neq a^2$

$\Rightarrow \sqrt{35n^2+42n+10}=\sqrt{7(5n^2+6n+1)+3}\in$ R-Q……………………………………………1p

b) Sa se arate ca numarul:

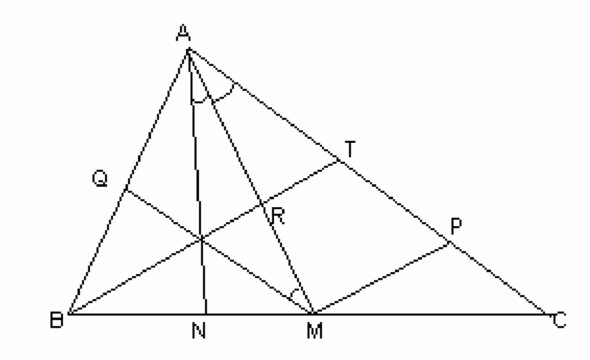
a= $\underbrace{22\ldots2}_{n\_cifre}{}^4-8\cdot\underbrace{22\ldots23}_{n\_cifre}{}^2+8\cdot\underbrace{44\ldots47}_{n\_cifre}$ este patrat perfect, oricare ar fi n$\in$ N

$$\left.\begin{array}{l}22\ldots23=22\ldots22+1=k+1\ldots\ldots\ldots\ldots1p\\44\ldots47=44\ldots44+3=2\cdot22\ldots22+3=2k+3\\22\ldots22=k\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots1p\end{array}\right\}\Rightarrow a=k^4+8(k+1)^2+8(2k+3)=(k^2-4)^2\ \ldots1p$$

2. In triunghiul ABC, BC = 2AB, AM este mediana (M$\in$ BC), N este mijlocul segmentului BM si P$\in$ AC , astfel incat MP $\perp$ AM.

a) Demonstrati ca AM este bisectoarea unghiului $N\hat{A}C$.

b) Calculati valoarea raportului $\frac{PC}{AP}$.



a) In ΔABM is. fie MQ mediana……………………………1p

Demonstram ca $M\hat{A}N \equiv A\hat{M}Q$ ………………………… 1p

MQ l.m. $\Rightarrow$ MQ || AC …………………………………….1p

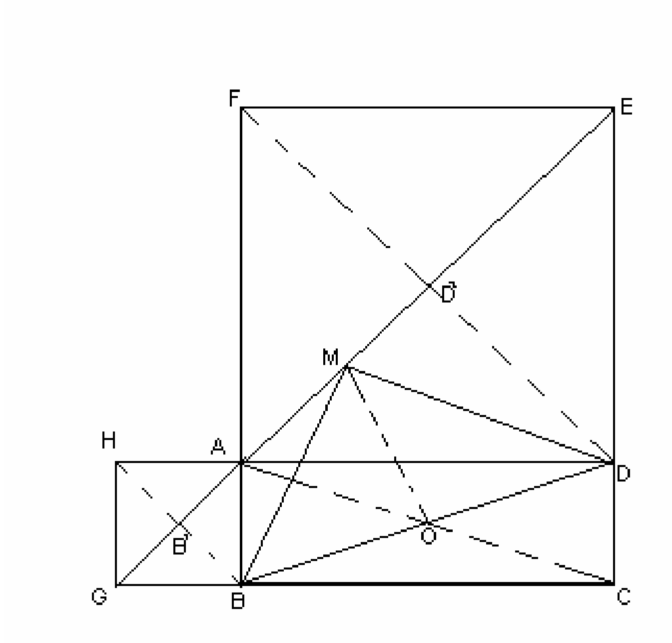
$\Rightarrow A\hat{M}Q \equiv M\hat{A}C \Rightarrow M\hat{A}N \equiv M\hat{A}C$ ………………………1p

b) ΔABM is. fie BR mediana $\Rightarrow \left.\begin{matrix} BR \perp AM \\ MP \perp AM \end{matrix}\right\} \Rightarrow BR \parallel MP$……………………………………….1p

BR $\cap$ AC={T}

$\left.\begin{matrix} \Delta AMP, RT l.m. \Rightarrow AT = TP ....1p \\ \Delta BTC, MP l.m. \Rightarrow TP = PC ....1p \end{matrix}\right\} \Rightarrow AT = TP = PC \Rightarrow \frac{PC}{AP} = \frac{1}{2}$……………………………… 1p

3. In exteriorul dreptunghiului ABCD se construiesc patratele ADEF, ABGH. Daca M este mijlocul segmentului GE, calculati masurile unghiurilor triunghiului MBD.



ΔGCE dr. is. $\Rightarrow$ CM $\perp$ GE$\Rightarrow$ ΔAMC dr……1p

Se demonstreaza ca G,A,E coliniare…………..1p

Fie AC$\cap$ BD = {O}$\Rightarrow$ OA=OB=OC=OD……..1p

$$\Rightarrow \text{OM}= \frac{AC}{2} = \frac{BD}{2} \Rightarrow \Delta MBDdr............1p$$

$$\left.\begin{array}{l} MD' = AB' = BB'.............1p \\ \text{B'D'}=\dfrac{GE}{2} \Rightarrow MB' = D'E = D'D.............1p \\ 9\hat{0} \end{array}\right\} \Rightarrow$$

$\Rightarrow$ ΔBB'M $\equiv$ ΔMDD $\Rightarrow$ MD = MB $\Rightarrow$

$\Rightarrow$ m $\widehat{MBD}$ = m $\widehat{MDB}$ = 45° …………………..1p